Simulation of industrial ceramic powder forming process through non-standard material models and multi-objective optimization

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The Extremely Flexible BP Yield Function

The BP yield function is defined in terms of the stress tensor $\boldsymbol{\sigma}$ by

$$F(\boldsymbol{\sigma}) = f(p) + \frac{q}{g(\theta)},$$

where, defining the parameter Φ as

$$\Phi = \frac{p+c}{p_c+c},$$

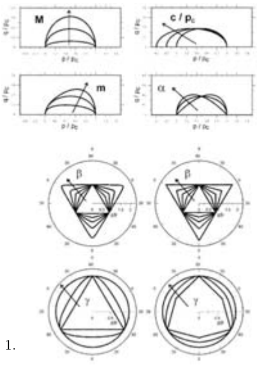
the meridian and deviatoric functions are respectively written as

$$f(p) = \begin{cases} -Mp_c\sqrt{(\Phi-\Phi^m)\left[2(1-\alpha)\Phi+\alpha\right]}, & \Phi \in [0,1], \\ +\infty, & \Phi \notin [0,1], \end{cases}$$

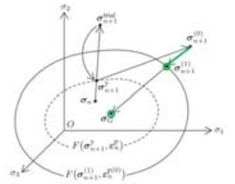
$$\frac{1}{g(\theta)} = \cos\left[\beta\frac{\pi}{6} - \frac{\cos^{-1}(\gamma\cos 3\theta)}{3}\right],$$

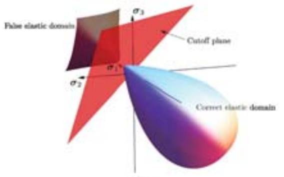
The yield function is convex when the seven material parameters defining the meridian shape function $f(p)$ and the deviatoric shape function $g(\theta)$ lie within the following intervals

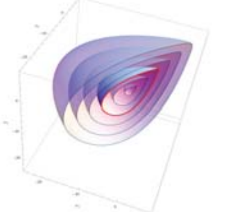
$$M>0, \quad p_c>0, \quad c\geq 0, \quad 0<\alpha<2, \quad m>1, \quad 0\leq\beta\leq 2, \quad 0\leq\gamma\leq 1.$$



New Integration Algorithms



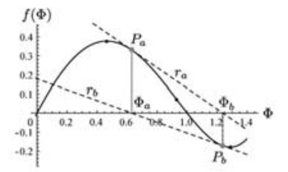




Used in the context of **elastoplastic modelling**, the BP yield function introduces the problem that for it to be convex, it has been defined $+\infty$ outside certain regions in the stress-space. Therefore, in its original form, the BP yield function cannot be implemented within an elastoplastic integration scheme, if a gradient-based return-mapping algorithm is used, for which the gradient of the yield function is needed everywhere in the stress-space.

To overcome the already mentioned difficulties of using standard integration procedures, we propose two new algorithms: one is based on a forward Euler technique with a correction based on a **'centre-of-mass' return scheme**, fully applicable to the original form of the BP yield function (defined $+\infty$ outside the yield surface), and another based on a **cutoff-substepping return-mapping algorithm** that can be applied on the squared (and non-convex) version of the BP yield function.

Another option is an **implicit definition of the BP yield function**, to avoid singularieties or wrong elastic domains certain regions in the stress-space. With this alternative formulation, the integration of the constitutive equations can be successfully performed with a standard fully implicit return-mapping procedure.

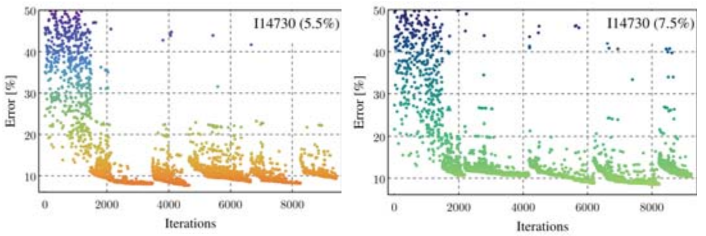


Parameter identification by multi-objective optimization

Some of the **material parameters**, involved in the constitutive model for ceramic powder densification, were identifed directly from the results of a set of experimental tests.
All the other material parameters, not identified directly through the mechanical tests, are determined by simulating the experiments and performing an iterative multi-objective optimization.

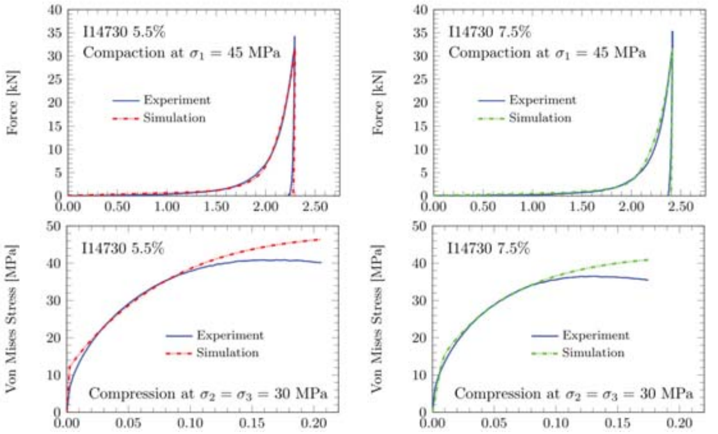
The optimization has been carried out with simplified numerical models, involving a small number of finite elements and neglecting the effects of friction.
This approach significantly **speed up** the FE simulations, so that the entire parameter identification could be performed on a simple laptop computer in a reasonable computational time.

As a sufficiently precise starting point was not available, both global and local optimization methods have been used to efficiently estimate the material parameters. The "**hybrid**" procedure involves first a Pareto optimization, by means of **moga** (multi-objective genetic algorithm). After a sufficiently high number of iterations of the global algorithm, the best five solutions are refined by a local optimization method (**pattern-search**).

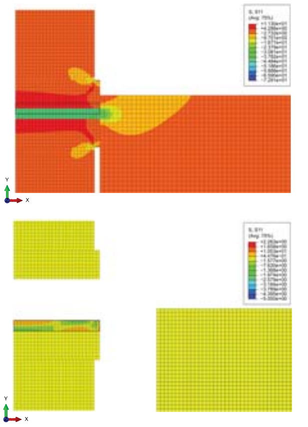


Comparison with experimental tests

The following figures show the comparison between numerical and experimental results, for both **uniaxial deformation tests** and **triaxial tests**.
With the identified parameters, it is possible to obtain a good agreement between numerical and experimental results also in tests which are not considered in the optimization. This is an indication of the **consistency of the constitutive model** and of the validity of the optimization procedure. The two figures on the top show the uniaxial compaction (force vs. displacement) curves for water content $w = 5.5\%$ (left) and $w = 7.5\%$ (right). The bottom figures show the compression/extension triaxial (von Mises stress vs. axial strain) curves for water content $w = 5.5\%$ (left) and $w = 7.5\%$ (right).



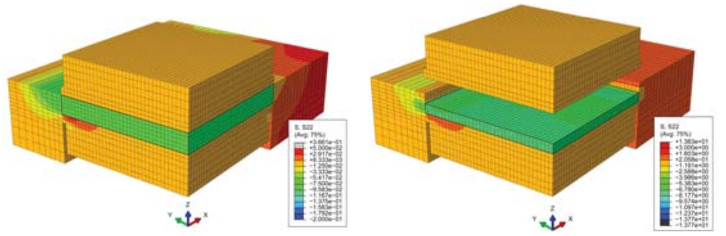
Simulation of industrial powder compaction processes



Finite element analyses, involving **contact** interaction and **friction**, have been performed to accurately reproduce the industrial processes of ceramic tablets and tiles forming. In these simulations, the complete forming device, composed by matrix, upper and lower punches, was modelled, and the interaction between each single part and the ceramic powder was taken into account.

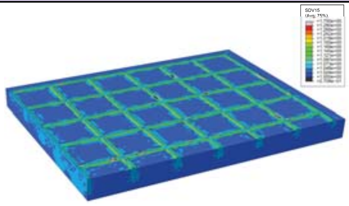
The simulation comprises the following steps: **geostatic step**, in which an initial confinement p_0 is imposed on the powder; **uniaxial compaction phase**, in which the upper punch is loaded by the given vertical pressure; **unloading step**, in which the load on the upper punch is removed; **extraction**, in which the matrix is removed in order to simulate the removal of the tablet from the cylindrical mould.

The stresses developing in the ceramic powder as well as in the mould are shown in the figures. The analysis made it also possible to investigate the influence of friction in the forming process and to determine the transversal pressure on the steel matrix.
The capabilities of the model to predict **residual stresses** are crucial, since residual stresses may lead to fracture of the green body, by end capping or lamination.



Density distribution in mesh-backed tiles after pressing

The **density distribution** in green bodies is of primary importance in the optimization of tile forming processes. In fact, non-uniform density can affect the subsequent sintering process, resulting in **poor quality** of the final ceramic product.
We notice higher void ratio values, corresponding to **lower density**, in correspondence of the protrusions. These results pave the way to more advanced **virtual prototyping** analyses, aiming at the optimization of the design of tile forming devices, in order to produce green bodies with improved density distribution.



References

[1] A. Piccolroaz et al., Yield criteria for quasibrittle and frictional materials: a generalization to surfaces with corners, Int. J. Solids Struct. 46 (2009).

[2] A. Piccolroaz et al., An elastoplastic framework for granular materials becoming cohesive through mechanical densification., Eur. J. Mech. A/Solids 25 (2006).

[3] J. Korelc, Multi-language and multi-environment generation of nonlinear finite element codes, Eng. with Comput. 18 (2002).

[4] M. Penasa et al., Integration algorithms of elastoplasticity for ceramic powder compaction., J. Eur. Cer. Soc. 34 (2014).

[5] S. Stupkiewicz et al., Implicit yield function formulation for granular and rock-like materials, Comput. Mech. 54 (2014).

[6] F. Bosi et al., Experimental investigation of the elastoplastic response of aluminum silicate spray dried powder during cold compaction, J. Eur. Ceram. Soc. 34 (2014).